1. $y = \pm \sqrt{2(\frac{1}{2}x^2 + 3/2)}$

2. $y = \pm \sqrt{2(-\frac{1}{2}x^2 + 25/2)}$

3. $y = x$

#1–14 all

4. $y = e^{x^2 + \ln 3}$ OR $y = 3e^{x^2}$

5. $y = e^{\frac{1}{2}x^2 + 2x + \ln 6}$ OR $y = 6e^{\frac{1}{2}x^2 + 2x}$

6. $y = \arctan x$

7. $y = -\ln(-(e^{\sin x} - 2))$

8. $y = \ln(e^x + 1)$

9. $y = \dfrac{1}{x^2 + 3}$

10. $y = (\ln x)^4$

11. $y = 100e^{1.5t}$ OR $y = e^{1.5t + \ln 100}$

12. $y = 200e^{-0.5t}$ OR $y = e^{-0.5t + \ln 200}$

13. $y = 50e^{kt}$ AND $y = e^{kt + \ln 100 - 5k}$

14. $y = 60e^{kt}$ AND $y = e^{kt + \ln 30 - 10k}$

Section 6.4 Exercises

In Exercises 1–10, use separation of variables to solve the initial value problem. Indicate the domain over which the solution is valid.

1. $\dfrac{dy}{dx} = \dfrac{x}{y}$ and $y = 2$ when $x = 1$
2. $\dfrac{dy}{dx} = -\dfrac{x}{y}$ and $y = 3$ when $x = 4$
3. $\dfrac{dy}{dx} = \dfrac{y}{x}$ and $y = 2$ when $x = 2$
4. $\dfrac{dy}{dx} = 2xy$ and $y = 3$ when $x = 0$
5. $\dfrac{dy}{dx} = (y + 5)(x + 2)$ and $y = 1$ when $x = 0$
6. $\dfrac{dy}{dx} = \cos^2 y$ and $y = 0$ when $x = 0$
7. $\dfrac{dy}{dx} = (\cos x)e^{y + \sin x}$ and $y = 0$ when $x = 0$
8. $\dfrac{dy}{dx} = e^{x-y}$ and $y = 2$ when $x = 0$
9. $\dfrac{dy}{dx} = -2xy^2$ and $y = 0.25$ when $x = 1$
10. $\dfrac{dy}{dx} = \dfrac{4\sqrt{y}\ln x}{x}$ and $y = 1$ when $x = e$

In Exercises 11–14, find the solution of the differential equation $dy/dt = ky$, k a constant, that satisfies the given conditions.

11. $k = 1.5$, $y(0) = 100$
12. $k = -0.5$, $y(0) = 200$
13. $y(0) = 50$, $y(5) = 100$
14. $y(0) = 60$, $y(10) = 30$

#1-6 in class

#7. $\frac{dy}{dx} = (\cos x)\, e^{y+\sin x}$

$\frac{dy}{dx} = \cos x \cdot e^{y} \cdot e^{\sin x}$

$\frac{1}{e^{y}}\, dy = \cos x \cdot e^{\sin x}\, dx$

$\int e^{-y}\, dy = \int \boxed{\cos x}\, e^{\overset{u}{\sin x}}\, \overset{du}{\boxed{dx}}$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x\, dx$

$\int e^{-y}\, dy = \int e^{u}\, du$

$-e^{-y} = e^{u} + c$

$-e^{-y} = e^{\sin x} + c$

$e^{-y} = -(e^{\sin x} + c)$

$\cancel{\ln}(e^{-y}) = \ln(-(e^{\sin x} + c))$

$-y = \ln(-(e^{\sin x} + c))$

$\boxed{y = -\ln(-(e^{\sin x} + c))}$

general solution

$0 = -\ln(-(e^{\sin 0} + c))$

$0 = -\ln(-(e^{0} + c))$

$0 = -\ln(-(1 + c))$

$0 = \cancel{-}\ln(-(1+c))$

$e \qquad e$

$1 = -(1+c)$

$-1 = 1 + c$

$c = -2$

particular solution

$\boxed{y = -\ln(-(e^{\sin x} - 2))}$

#8 $\frac{dy}{dx} = e^{x-y}$

$\frac{dy}{dx} = e^{x} \cdot e^{-y}$

$\frac{1}{e^{-y}}\, dy = e^{x}\, dx$

$\int e^{y}\, dy = \int e^{x}\, dx$

$e^{y} = e^{x} + c$

$\ln(e^{y}) = \ln(e^{x} + c)$

$\boxed{y = \ln(e^{x} + c)}$

general solution

$2 = \ln(e^{0} + c)$

$2 = \ln(1 + c)$

$e^{2} = e^{\ln(1+c)}$

$2 = \cancel{\ln}\ 1 + c$

$c = 1$

$\boxed{y = \ln(e^{x} + 1)}$

particular solution

#9 $\frac{dy}{dx} = -2xy^2$

$\frac{1}{y^2}\, dy = -2x\, dx$

$\int y^{-2}\, dy = \int -2x\, dx$

$-y^{-1} = -x^2 + c$

$y^{-1} = x^2 - c$

$\frac{1}{y} = x^2 - c$

$\boxed{y = \frac{1}{x^2 - c}}$

general solution

$0.25 = \frac{1}{(1)^2 - c}$

$0.25(1-c) = 1$

$1 - c = 1/0.25$

$1 - c = 4$

$c = -3$

$\boxed{y = \frac{1}{x^2+3}}$

particular sol.

#10 $\frac{dy}{dx} = \frac{4\sqrt{y}\ln x}{x}$

$\frac{1}{\sqrt{y}}\, dy = \frac{4 \ln x}{x}\, dx$

$\int y^{-1/2}\, dy = \int 4 \frac{\overset{u}{\ln x}}{x}\, dx$

$u = \ln x$

$du = \frac{1}{x}\, dx$

$\int y^{-1/2}\, dy = \int 4u\, du$

$2y^{1/2} = 2u^2 + c$

$2y^{1/2} = 2(\ln x)^2 + c$

$y^{1/2} = \frac{1}{2}(2(\ln x)^2 + c)$

$(y^{1/2})^2 = \left(\frac{1}{2}(2(\ln x)^2 + c)\right)^2$

$\boxed{y = \left((\ln x)^2 + \frac{1}{2}c\right)^2}$

general sol.

$1 = \left((\ln e)^2 + \frac{1}{2}c\right)^2$

$1 = \left(1 + \frac{1}{2}c\right)^2$

$\sqrt{1} = \sqrt{\left(1 + \frac{1}{2}c\right)^2}$

$1 = 1 + \frac{1}{2}c$

$c = 0$

$\boxed{y = \left[(\ln x)^2\right]^2} = \boxed{(\ln x)^4}$

particular sol.

$$\boxed{\frac{dy}{dt} = ky}$$

#11. $\frac{dy}{dt} = 1.5y$

$\int \frac{1}{y}\,dy = \int 1.5\,dt$

$e^{\ln|y|} = e^{1.5t + c}$

$\boxed{y = e^{1.5t+c}}$
gen. solution

$100 = e^{1.5(0)+c}$

$100 = e^{c}$

$c = \ln(100)$

$\boxed{y = e^{1.5t + \ln 100}}$
exact solution

OR

$\boxed{y = 100e^{1.5t}}$

#12 $\frac{dy}{dt} = -0.5y$

$\int \frac{1}{y}\,dy = \int -0.5\,dt$

$e^{\ln|y|} = e^{-0.5t + c}$

$\boxed{y = e^{-0.5t+c}}$
gen. sol.

$200 = e^{-0.5(0)+c}$

$200 = e^{c}$

$c = \ln(200)$

$\boxed{y = e^{-0.5t + \ln(200)}}$
exact sol.

OR

$\boxed{y = 200e^{-0.5t}}$

#13

$\frac{dy}{dt} = ky$

$\frac{1}{y} = k\,dt$

$\ln|y| = kt + c$

$\boxed{y = e^{kt+c}}$ gen. sol.

$(0,50)$

$50 = e^{0+c}$

$c = \ln 50$

$\boxed{y = e^{kt + \ln 50}}$

OR

$\boxed{y = 50e^{kt}}$

$(5,100)$

$100 = e^{5k+c}$

$\ln 100 = 5k + c$

$c = \ln(100) - 5k$

$\boxed{y = e^{kt + \ln(100) - 5k}}$

#14

$\frac{dy}{dt} = ky$

$\boxed{y = e^{kt+c}}$ gen. sol.

$(0,60)$

$60 = e^{k(0)+c}$

$60 = e^{0+c}$

$60 = e^{c}$

$c = \ln 60$

$\boxed{y = e^{kt + \ln 60}}$

OR

$\boxed{y = 60e^{kt}}$

$(10,30)$

$30 = e^{k(10)+c}$

$30 = e^{10k+c}$

$\ln 30 = 10k + c$

$c = \ln(30) - 10k$

$\boxed{y = e^{kt + \ln(30) - 10k}}$