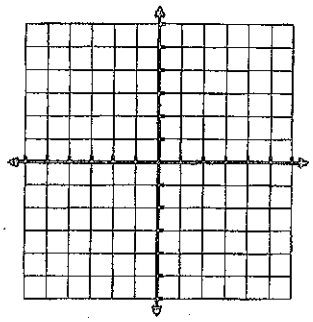
Unit 4 - Exponentials

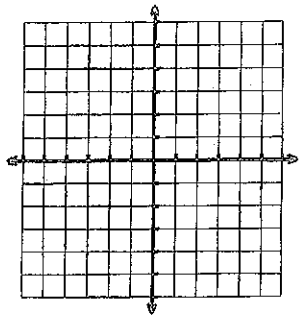
AC CCGPS Alg/Geo
Graphing Exponential Equations

Name _____

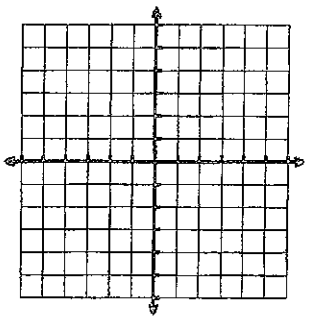
1. $y = 2^x$



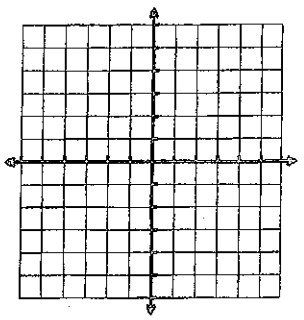
2. $y = (3)^x$



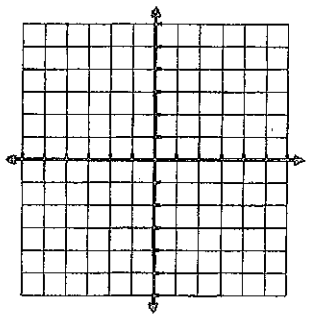
3. $y = 3^x + 2$



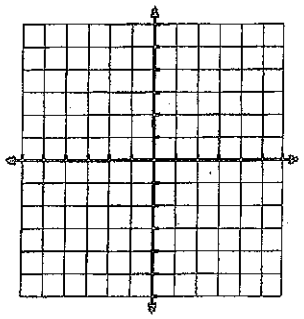
4. $y = 2^x + 1$



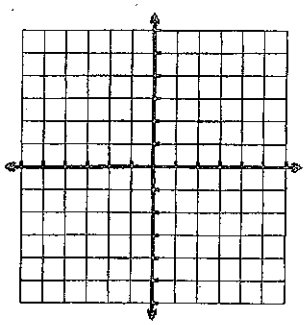
5. $y = -\left(\frac{1}{2}\right)^{x-2}$



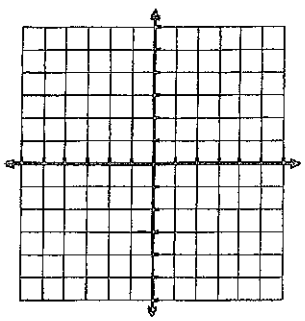
6. $y = 3^x - 2$



7. $y = \left(\frac{1}{3}\right)^x - 4$



8. $y = 2^x$



An exponential function has the form

$$y = ab^x$$

b is a positive number other than 1

If b is *greater than* 1

$y = ab^x$ ← b is the "growth factor"

exponential growth function.

If b is *between 0 and 1*

$y = ab^x$ ← b is the "decay factor"

exponential decay function.

In the bunny problem
a = 2 (because we initially had 2 bunnies)
b = 2 (because they were having 2 babies)
So our equation was $y = 2(2)^x$

a = initial amount
b = growth/decay factor
x = time
y = ending amount

In the meteorite problem:
a = 30 (we initially had 30 ounces)
b = ½ (it was decaying by ½)
So our equation was: $y = 30(\frac{1}{2})^x$

1. Each year the local country club sponsors a tennis tournament. Play starts with 128 participants. During each round, half of the players are eliminated. Write an equation that models this situation. _____
How many players are left after 5 rounds? _____ (you can count OR plug in 5 for your x)

2. Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. Write an equation that models one bacteria cell that splits into two new cells every hour. _____
How many bacteria would you have after 24 hours? _____

In the following equations, identify the initial amount and growth or decay factor. CIRCLE whether it is growth or decay.

3. $y = 100(3)^x$
Initial amount =
Growth/decay factor =

4. $y = 15(.5)^x$
Initial amount =
Growth/decay factor =

5. $y = 4^x$
Initial amount =
Growth/decay factor =

6. $y = \frac{1}{2}(\frac{3}{2})^x$
Initial amount =
Growth/decay factor =

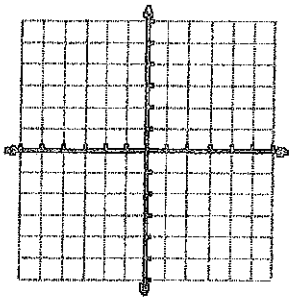
CCGPS A – Linear and Exponential Functions
Graphing Exponential Functions

Name _____

Exponential Functions: $y = b^x$, where b is a positive number other than 1

Graph $y = 2^x$ using a t-chart.

X	Y



Asymptote - a line that a graph approaches as you move away from the origin; the graph hugs the asymptote

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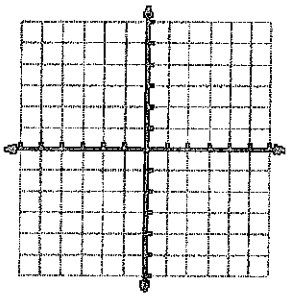
General Exponential Function $y = a(b^{x-h}) + k$

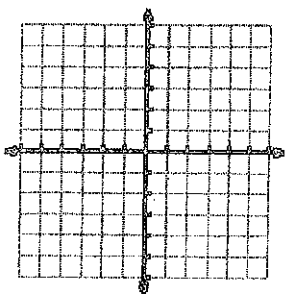
- Sketch the horizontal asymptote with a dashed line ($y = k$)
- Find the y-intercept of the graph by evaluating the function when $x=0$.
- Use a t-chart to sketch the graph of $y = ab^x$
- Transform the graph
  - Multiply y value of each coordinate in t-chart by a – move pencil to this point.
  - Shift h units horizontally
  - Shift k units vertically

1. $y = 2^x + 3$



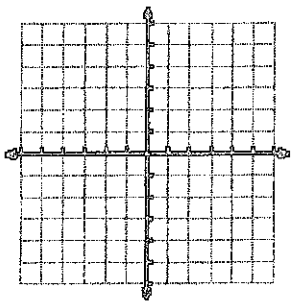
Y-intercept _____
Asymptote _____
Domain _____
Range _____
Growth or Decay
end behavior: as $x \to -\infty$, $y \to$ _____
as $x \to \infty$, $y \to$

2. $y = 2^{x+3} - 4$



Y-intercept _____
Asymptote _____
Domain _____
Range _____
Growth or Decay
end behavior: _____

3. $y = 3^{x-2}$



Y-intercept _____

Asymptote _____

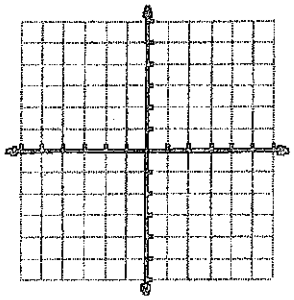
Domain _____

Range _____

Growth or Decay

end behavior: _____

4. $y = \left(\frac{1}{2}\right)^x + 3$



Y-intercept _____

Asymptote _____

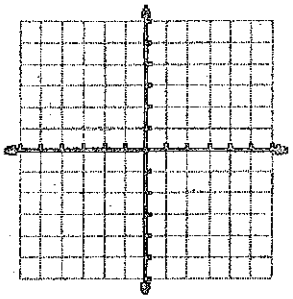
Domain _____

Range _____

Growth or Decay

end behavior: _____

5. $y = \left(\frac{1}{3}\right)^x - 2$



Y-intercept _____

Asymptote _____

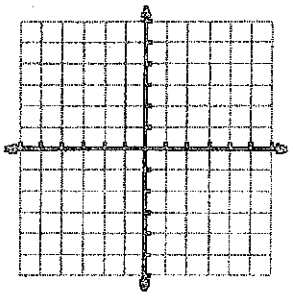
Domain _____

Range _____

Growth or Decay

end behavior: _____

6. $y = -(3)^x$



Y-intercept _____

Asymptote _____

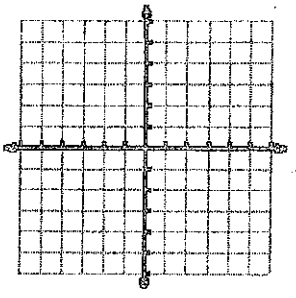
Domain _____

Range _____

Growth or Decay

end behavior: _____

7. $y = 3 \cdot (2)^x - 4$



Y-intercept _____

Asymptote _____

Domain _____

Range _____

Growth or Decay

end behavior: _____

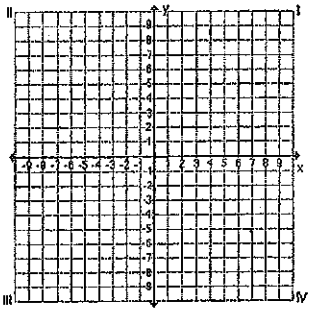
CCGPS A
Linear and Exponential Functions

Name _____

Graph the following by using transformations from the 'parent' graph. Graph 'parent points' in pencil and then apply transformation. Connect new points with curve.

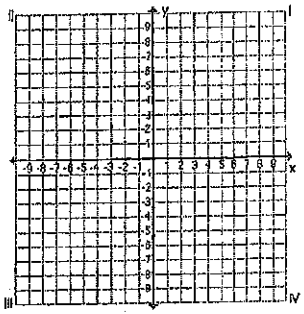
1. $y = 2^x - 4$

y – int ____
asymp ____
dom ____
range ____
growth/decay
e.b as $x \to -\infty, y \to$
as $x \to \infty, y \to$



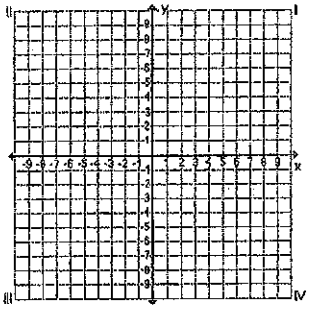
2. $y = \left(\frac{1}{2}\right)^x + 3$

y – int ____
asymp ____
dom ____
range ____
growth/decay
e.b.



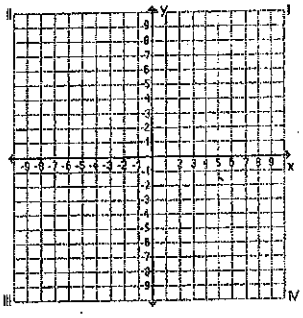
3. $y = \left(\frac{1}{3}\right)^x - 2$

y – int ____
asymp ____
dom ____
range ____
growth/decay
e.b.



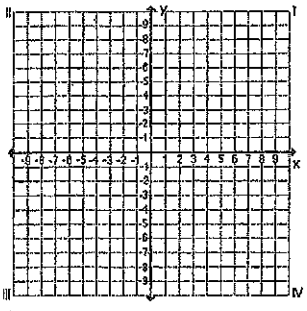
4. $y = 3^x - 2$

y – int ____
asymp ____
dom ____
range ____
growth/decay
e.b.



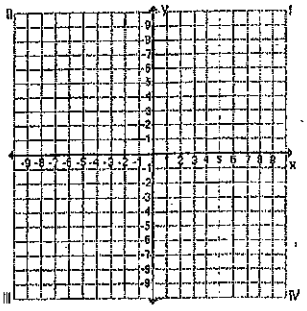
5. $y = 3\left(\frac{1}{3}\right)^x + 4$

y – int ____
asymp ____
dom ____
range ____
growth/decay
e.b.



6. $y = 2(2^x) - 3$

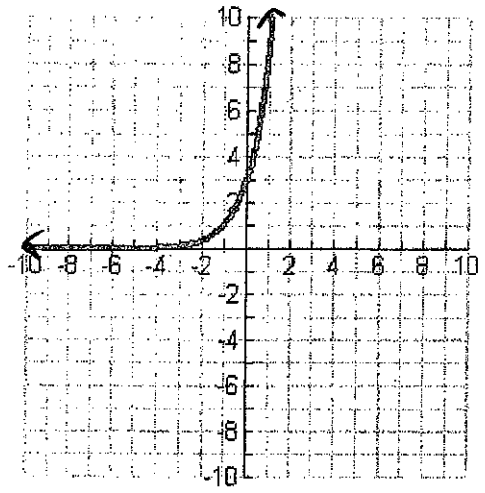
y – int ____
asymp ____
dom ____
range ____
growth/decay
e.b.



Name: _____ Homework

Analyzing Growth and Decay Exponential Functions

1. $y = 3^{x+1}$



Domain:

Range:

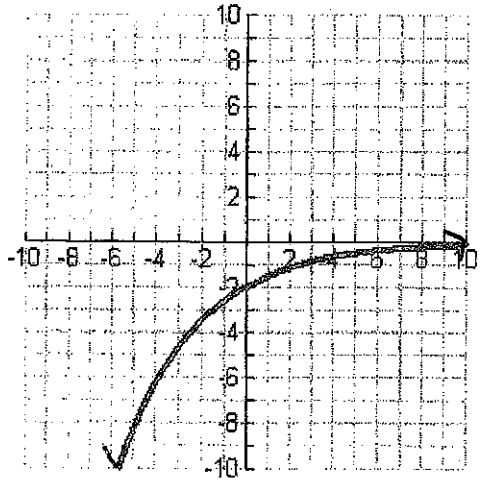
Asymptotes:

Zeros:

y-Intercepts:

Intervals of increase and decrease

2. $y = -2\left(\frac{3}{4}\right)^x$



Domain

Range:

Asymptotes:

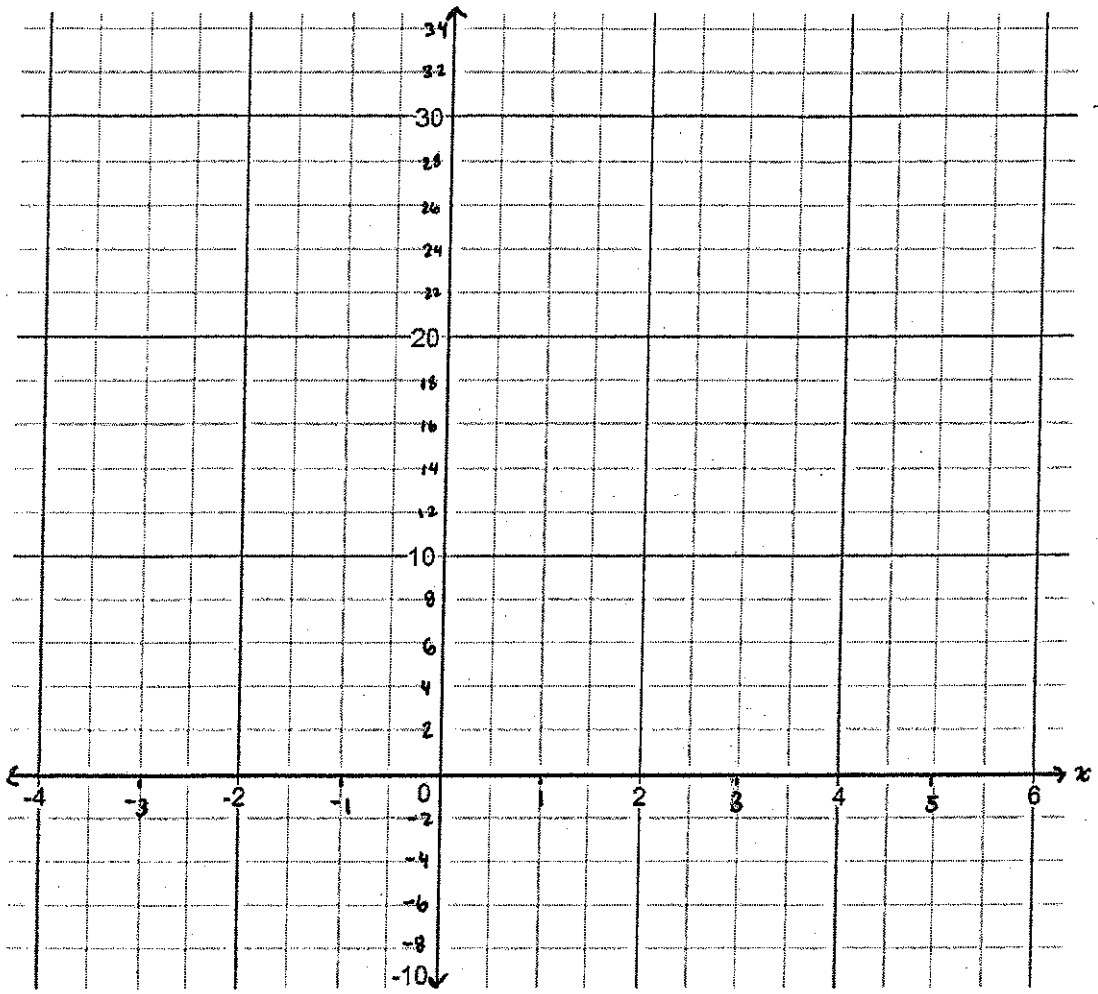
Zeros:

y-Intercepts:

Intervals of increase and decrease

EXPONENTIAL TRANSFORMATIONS

GRAPH the parent function $y = 2^x$



x	y
0	
1	
2	
3	
4	
5	

RED: Graph $y = 2^x + 6$ Transformation:

YELLOW: Graph $y = 2^x - 4$ Transformation:

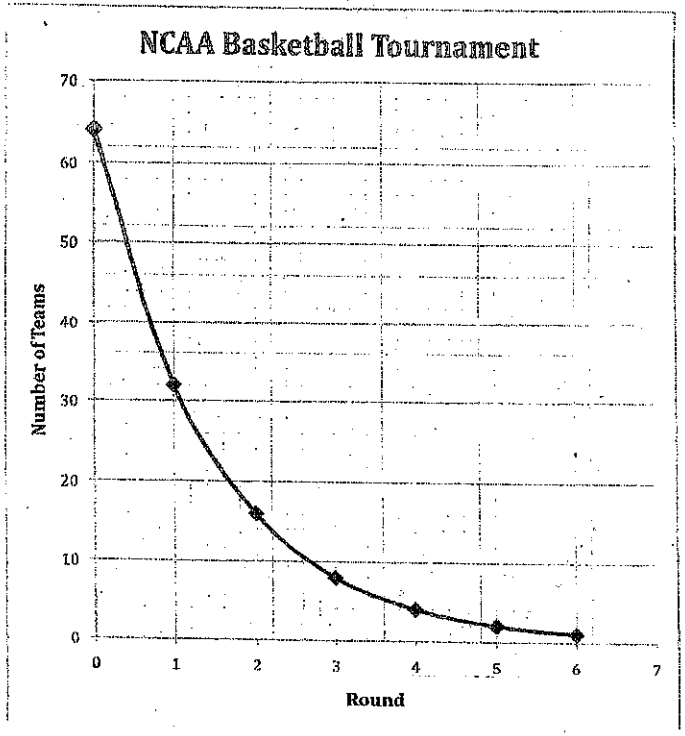
GREEN: Graph $y = 2^{x-4}$ Transformation:

BLUE: Graph $y = 2^{x+4}$ Transformation:

PURPLE: Graph $y = -2^x$ Transformation:

RATE OF CHANGE

Period: _____



Use the graph to answer the following questions.

1.) How many teams are there when the tournament starts?

2.) How many rounds occur before there is a winner?

3.) What is the rate of change between the 1st and 2nd round?

4.) What is the rate of change between the 2nd and 3rd round?

5.) What is the rate of change between the 3rd and 4th round?

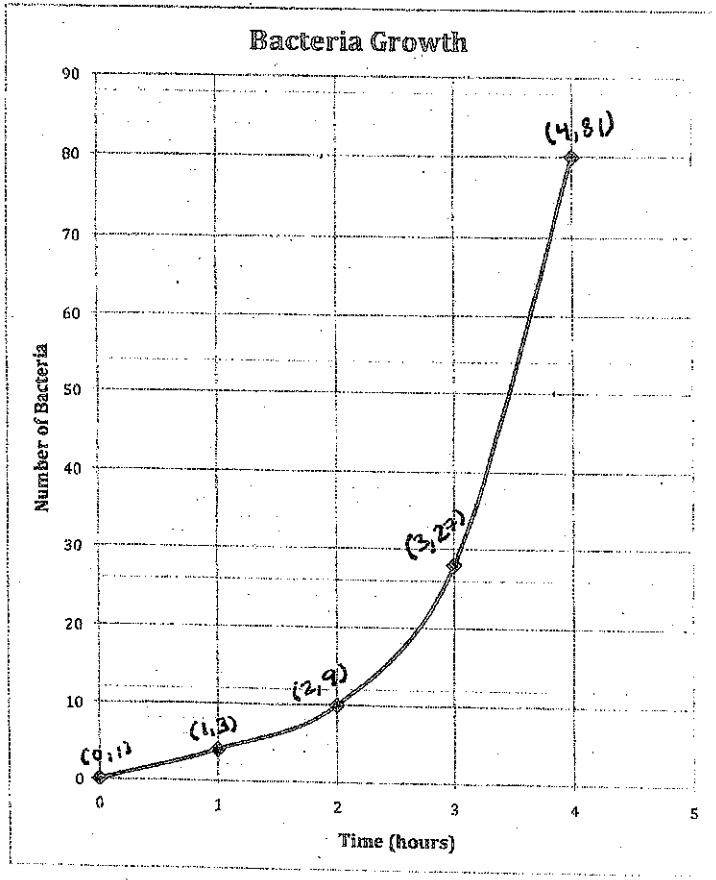
6.) What is the average rate of change between the 1st and 4th round?

7.) What is the average rate of change from the beginning of the tournament to the end?

8.) The NCAA tournament chairman is considering adding another round to the tournament so more teams can participate. How many teams would start the tournament?

Name: _____ Period: _____

The graph below shows the number of bacteria in a colony is growing.



Use the graph to answer the following questions.

14.) How many bacteria were there to start?

15.) What is the rate of change between 1 and 2 hours? $\frac{6 \text{ bacteria}}{1 \text{ hr}}$

-0

Name: _____ Period: _____

16.) What is the rate of change between 2 and 3 hours?

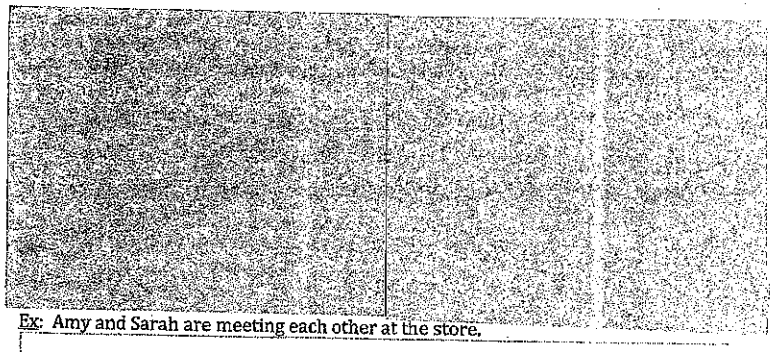
17.) Which interval of time has the biggest rate of change?

18.) What is the average rate of change between 1 and 3 hours?

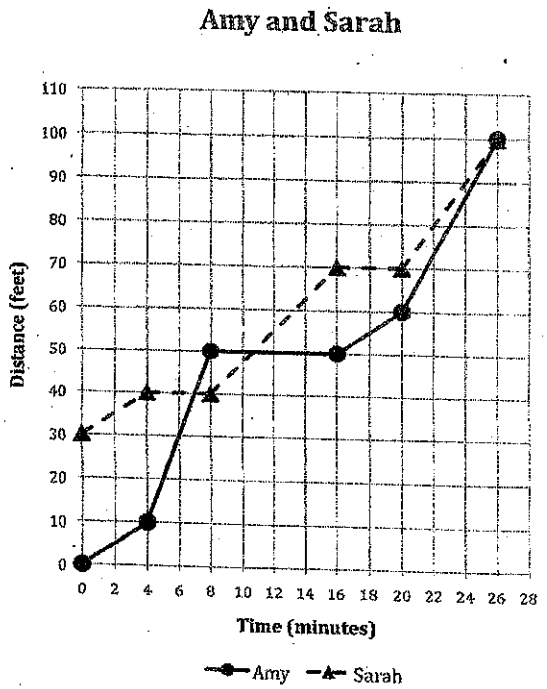
19.) What is the average rate of change from 0 to 4 hours?

Average Rate of Change

Notes:



Ex: Amy and Sarah are meeting each other at the store.



Name: _____ Period: _____

Use the graph to answer the following questions.

1.) How far from the store is Amy at the beginning?

2.) How far from the store is Sarah at the beginning?

3.) How long does it take to get to the store?

4.) What happens between 6 and 7 minutes?

5.) Where is Amy moving faster?

6.) Where is Sarah moving faster?

7.) What is the speed of Amy between 4 and 8 minutes?

8.) What is the speed of Sarah 8 and 16 minutes?

9.) What is Amy doing during 8 and 16 minutes?

10.) What is Amy's average speed for the whole trip?

11.) What is Sarah's average speed for the whole trip?

Notes - 3.4B Rate of Change

Ex: What is the average rate of change of the function $g(x) = 6 - 2x$

A.) Over the interval [2, 6]?

B.) Over the interval [5, 7]?

C.) Do you think it is true that $g(x)$ will have a constant average rate of change over *any* interval? Why or why not?

Ex: What is the average rate of change of the function $f(x) = 2^x$

A.) Over the interval [1, 4]?

B.) Over the interval [3, 5]?

C.) Do you think it is true that $f(x)$ will have a constant average rate of change over any interval? Why or why not?

Ex: Given a table, find the rate of change for each interval.

x	y
-3	4
-2	1
-1	0
0	1
1	4
2	9
3	16

A.) [0, 3]

B.) [-2, 1]

C.) [-3, -1]

ALGEBRA 2 WORKSHEET SOLVING EXPONENTIAL EQUATIONS

Solve each equation for x.

1) $2^x = 2^{3x-4}$

2) $3^{2x-1} = 3$

3) $25^{x+3} = 25^{5x-7}$

4) $4^{3x-5} = 4^{8-x}$

5) $5^{x+1} = 25$

6) $3^{x-5} = 27$

7) $2^{3x-4} = 8^{x-1}$

8) $3^{2x-4} = 1$

9) $4^{x+2} = 8$

10) $9^x = 27$

11) $\left(\frac{1}{3}\right)^{x+2} = 9^{3x}$

12) $\left(\frac{1}{4}\right)^{x-1} = 32^{x+3}$

13) $18^{4x} = 18^x$

14) $125^{3-2x} = 5^{x-1}$

15) $4^{x-1} = \frac{1}{64}$

16) $\left(\frac{1}{4}\right)^x = 8^{x-1}$

17) $3^x = 3\sqrt{3}$

18) $5^x = 25\sqrt{5}$

19) $4^{2x} = 16\sqrt[3]{4}$

20) $3^{x-4} = 9\sqrt{3}$

10.4 Practice - Exponential Functions

Solve each equation.

1) $3^{1-2n} = 3^{1-3n}$

3) $4^{2a} = 1$

5) $(\frac{1}{25})^{-k} = 125^{-2k-2}$

7) $6^{2m+1} = \frac{1}{36}$

9) $6^{-3x} = 36$

11) $64^{b} = 2^{5}$

13) $(\frac{1}{4})^{x} = 16$

15) $4^{3a} = 4^{3}$

17) $36^{3x} = 216^{2x+1}$

19) $9^{2n+3} = 243$

21) $3^{3x-2} = 3^{3x+1}$

23) $3^{-2x} = 3^{3}$

25) $5^{m+2} = 5^{-m}$

Geometric Sequence Notes

Geometric Sequences Worksheet

Determine whether each of the following sequences is arithmetic, geometric, or neither. Explain your decisions.

1) -4, 1, 6, 11, ...

2) 2, 8, 32, 128, ...

3) 1.5, 4.5, 13.5, 40.5, ...

For each of the following geometric sequences, find the common ratio. Then write the explicit formula for the sequence.

4) 10, 20, 40, 80, ...

5) 7, -7, 7, -7, ...

6) 3, -12, 48, -192, ...

7) 162, 108, 72, 48, ...

8) 100, 50, 25, 12.5, ...

9) Show work: What is the 14^{th} term of the geometric sequence: 3, 9, 27, 81, ...

10) Show work: What is the 11^{th} term of the geometric sequence: -2, 10, -50, 250, ...

11) Lidia's parents have offered her two different options to earn her allowance for a 9-week period over the summer. She can either get paid $30 each week, or $1 the first week, $2 the second week, $4 the third week, and so on.

a) Clearly explain if the second option forms a geometric sequence or not.

b) Show work and explain which option Lidia should choose.

12) Gabe and Erik are finding the 9^{th} term of the geometric sequence -5, 10, -20, ...

Is either of them correct? Explain.

Gabe

$r = \frac{10}{-5} = -2$

$a_9 = -5(-2)^{9-1}$

$= -5(512)$

$= -2560$

Erik

$r = \frac{10}{-5} = -2$

$a_9 = -5(-2)^{9-1}$

$= -5(-256)$

$= 1280$

Kuta Software - Infinite Algebra 2 Name_____

Geometric Sequences

Date_____ Period____

Determine if the sequence is geometric. If it is, find the common ratio.

1) −1, 6, −36, 216, ...

2) −1, 1, 4, 8, ...

3) 4, 16, 36, 64, ...

4) −3, −15, −75, −375, ...

5) −2, −4, −8, −16, ...

6) 1, −5, 25, −125, ...

Given the explicit formula for a geometric sequence find the first five terms and the 8th term.

7) $a_n = 3^{n-1}$

8) $a_n = 2 \cdot \left(\frac{1}{4}\right)^{n-1}$

9) $a_n = -2.5 \cdot 4^{n-1}$

10) $a_n = -4 \cdot 3^{n-1}$

Given the recursive formula for a geometric sequence find the common ratio, the first five terms, and the explicit formula.

11) $a_n = a_{n-1} \cdot 2$
$a_1 = 2$

12) $a_n = a_{n-1} \cdot -3$
$a_1 = -3$

13) $a_n = a_{n-1} \cdot 5$
$a_1 = 2$

14) $a_n = a_{n-1} \cdot 3$
$a_1 = -3$

Given the first term and the common ratio of a geometric sequence find the first five terms and the explicit formula.

15) $a_1 = 0.8,\ r = -5$

16) $a_1 = 1,\ r = 2$

Given the first term and the common ratio of a geometric sequence find the recursive formula and the three terms in the sequence after the last one given.

17) $a_1 = -4,\ r = 6$

18) $a_1 = 4,\ r = 6$

19) $a_1 = 2,\ r = 6$

20) $a_1 = -4,\ r = 4$

Given a term in a geometric sequence and the common ratio find the first five terms, the explicit formula, and the recursive formula.

21) $a_4 = 25,\ r = -5$

22) $a_1 = 4,\ r = 5$

Given two terms in a geometric sequence find the 8th term and the recursive formula.

23) $a_4 = -12$ and $a_5 = -6$

24) $a_5 = 768$ and $a_2 = 12$

25) $a_1 = -2$ and $a_5 = -512$

26) $a_5 = 3888$ and $a_3 = 108$

$y = P(1 \pm r)^t$

GPS Advanced Algebra Unit 3 Name: _____ Pd: ____

Exponential Growth and Decay Word Problems

1. Find a bank account balance if the account starts with $100, has an annual rate of 6%, and the money left in the account for 12 years.

2. In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers increased by 7% per year after 1985. How many cell phone subscribers were in Centerville in 1994?

3. Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. If we start with only one bacteria which can double every hour, how many bacteria will we have by the end of one day?

4. Each year the local country club sponsors a tennis tournament. Play starts with 128 participants. During each round, half of the players are eliminated. How many players remain after 8 rounds?

5. The population of Winnemucca, Nevada, can be modeled by $P=6191(1.04)^t$ where t is the number of years since 1990. What was the population in 1990? By what percent did the population increase by each year?

6. You have inherited land that was purchased for $30,000 in 1960. The value of the land increased by approximately 3% per year. What is the approximate value of the land in the year 2011?

7. During normal breathing, about 10% of the air in the lungs is replaced after one breath. Write an exponential decay model for the amount of the original air left in the lungs if the initial amount of air in the lungs is 500 mL. How much of the original air is present after 240 breaths?

8. An adult takes 400 mg of ibuprofen. Each hour, the amount of ibuprofen in the person's system decreases by about 29%. How much ibuprofen is left after 6 hours?

9. You deposit $1600 in a bank account. Find the balance after 3 years for each of the following situations:
   a. The account pays 3.5% annual interest compounded monthly.

   b. The account pays 1.75% annual interest compounded quarterly.

   c. The account pays 8% annual interest compounded yearly.

10. You buy a new computer for $2100. The computer decreases by 50% annually. When will the computer have a value of $600?

11. You drink a beverage with 120 mg of caffeine. Each hour, the caffeine in your system decreases by about 10%. How long until you have 10mg of caffeine?

12. The foundation of your house has about 1,200 termites. The termites grow at a rate of about 3.4% per day. How long until the number of termites doubles?

Compound Interest Worksheets

Name_____

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

Calculate the total amount of the investment or total paid in a loan in the following situations:

1.) Your 3 year investment of $20,000 received 5.2% interested compounded semi annually. What is your total return?

Answer:

2.) You borrowed $59,000 for 2 years at 11% which was compounded annually. What total will you pay back?

Answer:

3.) Your allowance of $190 got 11% compounded monthly for 1 2/3 years. What's it worth after the 1 2/3 years?

Answer:

4.) Your 6 1/4 year investment of $40,000 at 14% compounded quarterly is worth how much now?

Answer:

5.) You borrowed $1,690 for 5 1/2 years a at 5.7% compounded semi annually. What total will you pay back?

Answer:

6.) Your $440 gets 5.8% compounded annually for 8 years. What will your $440. be worth in 8 years?

Answer:

7.) Your $54,200 2 year car loan is at 15.1% compounded annually. What will you have paid for your car after 2 years?

Answer:

8.) You invest $55 at 10% compounded annually for 3 years. How much will your investment be worth in 3 years?

Answer:

9.) Your 8 year loan of $12,200 is at 5.3% compounded annually. How much will you have paid in total for your loan?

Answer:

10.) You invest $1,900 at 4% and it's compounded semi annually for 3 years. How much will your $1,900 be worth in 3 years?

Answer: